Microrheology of Complex Fluid

- Rheology: Science of the deformation & flow of matter
- Microrheology
  - Microscopic scale samples
  - Micrometer lengths

Complex shear modulus $G^*(\omega)$

$$\sigma = G^* \varepsilon$$

- $G^*(\omega) = G'(\omega) + jG''(\omega)$
- Solid vs. fluid
- Resistance to deformation

Storage modulus $G'$
Energy storage
Elasticity ~ Solid

Loss modulus $G''$
Energy dissipation
Viscosity ~ Fluid
High Frequency Microrheology Measurement

Active Method:
- Magnetic microrheometer – Baush, BJ 1998
  Huang, BJ 2002

Passive Method:
- Single particle tracking – Mason, PRL 1995
  Yamada, BJ 2000
- Multiple particle tracking – Crocker, PRL 2000
Magnetic Microrheology

5 sec Step Response
Ferromagnetic particle

\[ F = \frac{1}{2} \mu_0 \nabla (m \cdot H) \]

Paramagnetic particle – no permanent magnetic moment

\[ F = \mu_0 \chi \nabla \nabla (H \cdot H) \]

\( \chi \) is susceptibility

\( V \) is volume

Note: (1) force depends on volume of particle
(5 micron bead provide 125x more force)
(2) force depends on magnetic field GRADIENT

Particles cluster together!
Doesn’t work!
Magnetic manipulation in 3D

Top View

- ST: stage
- PH: post holder
- CO: coil
  - (400 turn/cm)
- PO: pole
- SA: sample chamber
- OB: objectives
  - (100x, 1.0 n.a.
  - water;
  - 20x, 0.5 n.a
  - all reflecting)

Side View

*Lower Force
nN level

*3D

*Uniform gradien

Amblad, RSI 1996
Huang, BJ 2002
Magnetic manipulation in 1D

*High force >10 nN

*Field non-uniform
Needs careful alignment of tip to within microns

*1D

Baush, BJ 1998

The bandwidth of ALL magnetic microrheometer is limited by the inductance of the electromagnet to about kiloHertz.
Magnetic Rheometer Requires Calibration

Baush, BJ 1998
Mag Rheometer Experimental Results

Baush, BJ 1998

Transient responses allow fitting to micro-mechanical model

Problem – Magnetic bead rolling

Solution – Injection, Endocytosis Modeling (Karcher BJ 2003)
Model Strain Field Distribution

Baush, BJ 1998
Consider the thermal driven motion of a sphere in a complex fluid.

**Langevin Equation**

\[ m\dot{v}(t) = f(t) + \int_{0}^{t} \xi(t-t')v(t')dt' \]

- Inertial force
- Random thermal force
- Memory function—Material viscosity
- Particle shape
Langevin Equation in Frequency Domain

Laplace transform of Langevin Equation

\[ \tilde{\nu}(s) = \frac{\tilde{f}(s) + mv(0)}{\tilde{\xi}(s) + ms} \]

Multiple by \( v(0) \), taking a time average, ignoring inertial term

Random force
\[ < \tilde{f}(s)v(0) > = 0 \]

Equipartition of energy
\[ m < v(0)v(0) > = kT \]

Generalized Stokes Einstein
\[ \tilde{\xi}(s) = 6\pi a \tilde{\eta}(s) \quad \tilde{G}(s) = s \tilde{\eta}(s) \]

Definition and Laplace transform of mean square displacement
\[ < v(0)\tilde{\nu}(s) > = \frac{s^2 < \Delta \tilde{r}^2(s) > }{6} \]
(2) Fluorescence Laser Tracking Microrheometer

- Approach: Monitoring the Brownian dynamics of particles embedded in a viscoelastic material to probe its frequency-dependent rheology

\[ \langle \Delta R^2(\tau) \rangle = \langle (\mathbf{R}(t+\tau) - \mathbf{R}(t))^2 \rangle \]

\[ G^*(i\omega) = \frac{2k_B T}{3\pi \cdot a \cdot i\omega \cdot \langle \Delta \mathbf{R}^2(i\omega) \rangle} \]

(2) Nanometer Resolution for the Bead’s Trajectory

- Collecting enough light from a fluorescent bead is critical

\[
p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(x - x_c)^2}{2\sigma^2} \right]
\]

\[
\frac{N_A}{N_B} = \frac{p(A)}{p(B)} = \frac{\int_{-\infty}^{x_c} \exp \left( -\frac{x^2}{2\sigma^2} \right) dx}{\int_{x_c}^{\infty} \exp \left( -\frac{x^2}{2\sigma^2} \right) dx}
\]

<table>
<thead>
<tr>
<th>Photons detected per measurement</th>
<th>(10^3)</th>
<th>(10^4)</th>
<th>(10^5)</th>
<th>(10^6)</th>
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<tbody>
<tr>
<td>Uncertainty on (\frac{N_A}{N_B})</td>
<td>0.033</td>
<td>0.010</td>
<td>0.003</td>
<td>0.001</td>
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<tr>
<td>Uncertainty on (x_c) (nm)</td>
<td>12</td>
<td>4</td>
<td>1.2</td>
<td>0.4</td>
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Nanometer resolution \(\leftrightarrow\) \(10^4\) photons per measurement
(2) Calibrating the FLTM

- 5-nm stepping at 5 or 50 kHz
- Curve fitting matches theory

![Graphs showing 5-nm steps at 5 kHz and 50 kHz, with corresponding ratio plots and fitting equations.](image)
Characterizing the FLTM

- Using polyacrylamide gels (w/v 2% to 5%) of known properties
- Good agreement with previously published data

Schnurr B., Gittes F., MacKintosh F.C. & Schmidt C.F
Single Particle Tracking Data

Yamada BJ 2000
SPT responses can be influenced by local processes (adhesion, active, etc) and not represent global cytoskeleton behavior.

Solution: Look at the correlated motion of two particles under thermal force.

\[
D_{rr}(r, \tau) = \langle \Delta r^i_r(t, \tau) \Delta r^j_r(t, \tau) \delta(r - R^{ij}(t)) \rangle_{i \neq j, t}
\]

\[
D_{rr}(r, s) = \frac{kT}{2\pi rs \tilde{G}(s)}
\]

The major difference is that the correlation signal is a function of “r” the separation of the particles but not their size.

Instead of using fast quadrant detectors, multiple particle tracking uses a wide field camera which is slower.
SPT vs MPT

Triangle: SPT

Circle: MPT

SPT and MPT results can be quite different specially in cells

Crocker, PRL 2000
<table>
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<tr>
<th></th>
<th>Magnetic</th>
<th>SPT</th>
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<tr>
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<td>Simple</td>
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